

## 3 Exponents, Logarithms, and Their Graphs

10/13/14

### 3.1 Geometric Sequences (Review)

Complete your Unit 2 test.

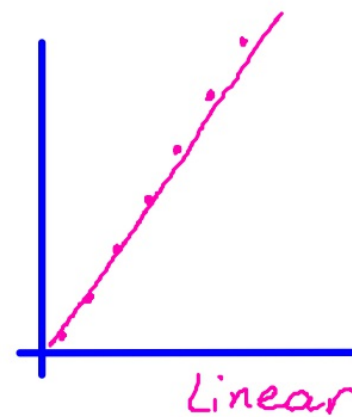
### 3.1 Geometric Sequences (Review)

10/13/14

**Recall arithmetic sequences**

$$a_n = a_{n-1} + d$$

$\nearrow$  previous term       $\uparrow$  common difference



Find  $a_8$  given the following sequence:

1, 5, 9, 13, ...

$d=4$       17, 21, 25, (29)



### 3.1 Geometric Sequences (Review)

10/13/14

IWBAT define geometric sequence and identify and describe the key characteristics of geometric sequences. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

### 3.1 Geometric Sequences (Review)

Recall geometric sequences

$$a_n = r * a_{n-1} \quad r = \text{common ratio}$$

Find  $a_8$  given the following sequence:

1, 4, 16, ...

$$r = 4 \quad 64, 256, 1024, 4096, 16384$$

$a_4 \quad a_5 \quad a_6 \quad a_7 \quad a_8$



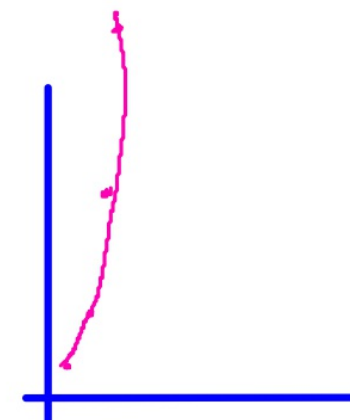
Recall shifted geometric sequences

$$a_n = r * a_{n-1} + c = r(a_{n-1} + d)$$

$$y = mx + b$$

↑                      ↑  
Slope                  y-intercept

+ |  
- |



END

IWBAT define geometric sequence and identify and describe the key characteristics of geometric sequences.

### 3.2 Exponential Functions

10/14/14

How does a geometric sequence differ from an arithmetic sequence?

arithmetic

linear

add/subtract

geometric

non-linear (curved)

Multiply/divide

## 3.2 Exponential Functions

10/14/14

**Define exponential growth, exponential decay, and compound interest.**

Exponential growth - a quantity grows exponentially over time

$$f(x) = 3^x$$

$$g(x) = 1.2^x$$

$$h(x) = \frac{9}{5}^x$$

base  $> 1$



$$f(x) = b^x$$

Exponential decay - a quantity decays exponentially over time

$$f(x) = 0.3^x$$

$$g(x) = .12^x$$

$$h(x) = \frac{5}{9}^x$$

base  $< 1$



## 3.2 Exponential Functions

10/14/14

Define exponential growth, exponential decay, and compound interest.

Compound interest - the accrual of interest in an account when the interest gained also collects interest

Sam puts \$300 in a savings account which pays 1.9% annually compounded monthly.

annually - 1 time per year

monthly - 12

weekly - 52

daily - 365



## 3.2 Exponential Functions

10/14/14

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

## 3.2 Exponential Functions

10/14/14

Identify the general formula for an exponential function.

$$f(x) = a * b^x$$

$a$  = initial amount

$b$  = growth factor

$$f(x) = 2 * 3^x$$

$$a = 2$$

$$b = 3$$

$$f(x) = .5^x$$

$$a = 1$$

$$b = 0.5$$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

## 3.2 Exponential Functions

10/14/14

Evaluate an exponential function for a given input value.

Find  $f(3)$

$$f(x) = 3^x$$

$$f(3) = 3^3 = 27$$

$$f(x) = 2 * 3^x$$

$$f(3) = 2 * 3^3 = 54$$

$$f(x) = 3 * .5^x$$

$$f(3) = 3 * .5^3 = 3 * 0.125 =$$

$$f(3) = 3 \left(\frac{1}{2}\right)^3 = 3 \left(\frac{1}{8}\right) = \frac{3}{8} = 0.375$$

$$f(x) = \left(\frac{1}{3}\right) * 25^x$$

$$f(3) = \left(\frac{1}{3}\right) \left(\frac{1}{4}\right)^3 = \left(\frac{1}{3}\right) \left(\frac{1}{64}\right) = \frac{1}{192}$$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.



### 3.2 Exponential Functions

10/14/14

Find the value of an account when given the initial investment amount, interest rate, compounding period, and time.

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

principle =  $P$

interest rate =  $r$

time =  $t$

$A(t)$  = principle after  
the interest is added

number of compounding  
periods =  $n$

FV = future value

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

## 3.2 Exponential Functions

10/14/14

Find the value of an account when given the initial investment amount, interest rate, compounding period, and time.

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

Sam puts \$300 in a savings account which pays 1.9% compounded annually. How much is in his account after 3 years?

$$A(3) = 300\left(1 + \frac{0.019}{1}\right)^{1 \cdot 3} = 300(1.019)^3$$

$$A(3) = \$317.42$$

Sam puts \$300 in a savings account which pays 1.9% annually compounded monthly. How much is in his account after 3 years?

$$A(3) = 300\left(1 + \frac{0.019}{12}\right)^{12 \cdot 3}$$

$$A(3) = \$317.58$$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

## 3.2 Exponential Functions

10/14/14

Find the value of an account when given the initial investment amount, interest rate, compounding period, and time.

$e$  is an irrational number. This means that its decimal representation goes on and on forever and does not repeat.

$$e \approx 2.7182818284590\dots$$

$e$  is approximated by  $\left(1 + \frac{1}{n}\right)^n$  for very large values of  $n$

$$n > 300$$

For continuous compounding:

$$A(t) = Pe^{rt}$$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.



## 3.2 Exponential Functions

10/14/14

Find the value of an account when given the initial investment amount, interest rate, compounding period, and time.

Sam puts \$300 in a savings account which pays 1.9% annually compounded continually. How much is in his account after 3 years?

$$A(t) = Pe^{rt}$$

$$300 e^{(0.019 \times 3)} = \$317.59$$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

## 3.2 Exponential Functions

10/14/14

Vocabulary 3.2.1 p. 22

Practice 3.2.2

Apex quizzes 3.2.3 & 3.2.4

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

## 3.2 Exponential Functions

10/15/14

Define exponential growth, exponential decay, and compound interest in your own words.

exponential growth  
exponential decay

interest gained in an account / on money  
(exponential growth)

$$A(t) = P \left( 1 + \frac{r}{n} \right)^{nt}$$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

## 3.2 Exponential Functions

10/15/14

### Practice 3.2.2

$$3) F(4) \quad f(x) = 9\left(\frac{1}{5}\right)^x$$

$$f(4) = 9\left(\frac{1}{5}\right)^4 = 9\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right) = \frac{9}{625}$$

$$9\left(\frac{1^4}{5^4}\right) = \frac{9}{5^4}$$

$$2) f(2) = \frac{1}{8} 5^2 = \frac{1}{8} 25 = \frac{25}{8} = 3\frac{1}{8}$$

$$5) A(6) = 400\left(1 + \frac{0.05}{1}\right)^{6.1}$$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

## 3.2 Exponential Functions

10/15/14

### Practice 3.2.2

8)  $P = \$100$   $A(t) = Pe^{rt}$   $A(6) = 100e^{(0.05 \cdot 6)}$   
 $r = 5\%$   
 $t = 6 \text{ yr}$   
 $C/y = \text{cont.}$

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.



## 3.2 Exponential Functions

10/15/14

Vocabulary 3.2.1 p. 22

Practice 3.2.2

Apex quizzes 3.2.3 & 3.2.4

IWBAT identify the general formula for an exponential function, find the value of an account when given the initial investment amount, interest rate, compounding period, and time, and evaluate an exponential function for a given input value.

### 3.3 Examples of Applications of Exponential Functions

10/16/14

Evaluate an exponential function for a given input value.

Find  $f(4)$ .

$$f(x) = .5 * 3^x \quad f(4) = .5 * 3^4 \\ .5 * 81 \quad f(4) = 40.5 \text{ or } \frac{81}{2}$$

$$f(x) = 12^x \quad f(4) = 12^4 = 20736$$

$$f(x) = 2 * \left(\frac{1}{3}\right)^x \quad f(4) = 2 * \left(\frac{1}{3}\right)^4 \\ = \frac{2}{81}$$

### 3.3 Examples of Applications of Exponential Functions

10/16/14

**Find the value of an account when given the initial investment amount, interest rate, compounding period, and time.**  $A(t) = P \left(1 + \frac{r}{n}\right)^{nt}$

\$400, 2.2%, compounded monthly, 5 years

$$\begin{aligned} \$ & A(5) = 400 \left(1 + \frac{0.022}{12}\right)^{12 \cdot 5} = \$446.46 \end{aligned}$$

\$3,000, 8.8% per day, 14 days

$$\begin{aligned} A(14) &= 3000 \left(1 + \frac{0.088}{1}\right)^{14} \\ A(14) &= \$9770.70 \end{aligned}$$

### 3.3 Examples of Applications of Exponential Functions

10/16/14

IWBAT define logistic growth and use exponential functions to solve real-world problems. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

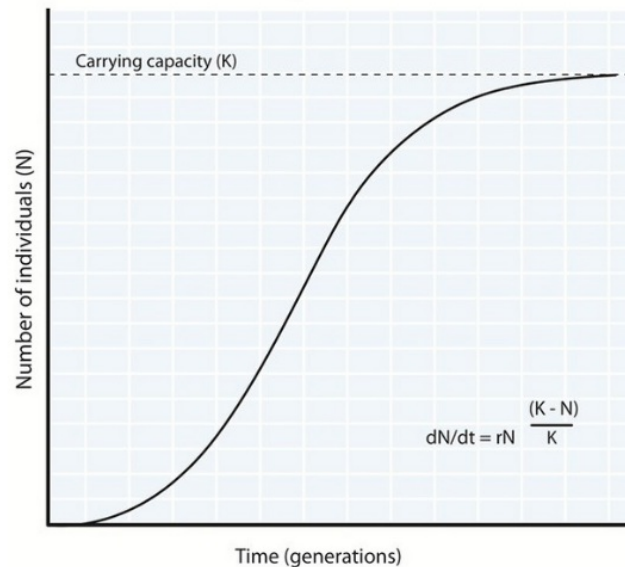
### 3.3 Examples of Applications of Exponential Functions

10/16/14

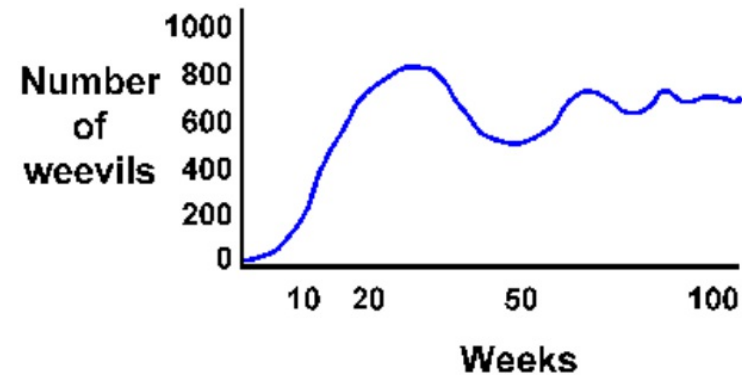
Logistic Growth: A type of growth that is exponential early, but slows as it reaches a maximum value.

#### Idealized

Logistic Growth



#### Realistic



IWBAT define logistic growth and use exponential functions to solve real-world problems.



### 3.3 Examples of Applications of Exponential Functions

10/16/14

Carbon Dating Problem  
Apex 3.3.1, pp. 8-10

$$C(t) = e^{-0.000121t}$$

$$C(t) = 2.1\%$$

$$\begin{aligned} 0.021 &= e^{-0.000121t} \\ \ln(0.021) &= \ln(e^{-0.000121t}) \\ \frac{\ln(0.021)}{-0.000121} &= \frac{-0.000121t}{-0.000121} \\ t &\approx 31,927 \end{aligned}$$

IWBAT define logistic growth and use exponential functions to solve real-world problems.

### 3.3 Examples of Applications of Exponential Functions

10/16/14

#### The Recycling Problem

In 1960, less than 1% of the families in Newtown recycled. A new project was started to encourage more recycling. Since then, the percentage of recycling families has been growing according to logistic growth. What percentage of families were recycling in 1980? What percentage of families will be recycling in 2040?

$$P(t) = \frac{100}{1 + 400 * e^{(-0.15t)}} \quad t = \text{years since 1960}$$

2040  
 $100 / (1 + 400 e^{(-0.15 * 80)}) = 95.29 \text{ } 99.75\%$

1980  
 $2040 - 1960 = 80$   
 $P(20) = \frac{100}{1 + 400 * e^{(-0.15 * 20)}} = 4.78\%$

IWBAT define logistic growth and use exponential functions to solve real-world problems.

## 3.3 Examples of Applications of Exponential Functions

10/16/14

Vocabulary 3.3.1 p. 15

Practice 3.3.2

IWBAT define logistic growth and use exponential functions to solve real-world problems.



### 3.4 Graphs of Exponential Functions

10/17/14

Find the value of an account when given the initial investment amount, interest rate, compounding period, and time.

\$20,000 , 0.01%, monthly, 19 years

$$A(19) = 20,000 \left(1 + \frac{0.0001}{12}\right)^{12 \cdot 19} = \$20038.03$$

\$1200, 1.3%, continuously, 5 years

$$A(5) = 1200 e^{0.013 \cdot 5} = \$1280.59$$

### 3.4 Graphs of Exponential Functions

10/17/14

**Identify the y-intercept of an exponential function given in the form  $F(x) = a \cdot b^x$ .**

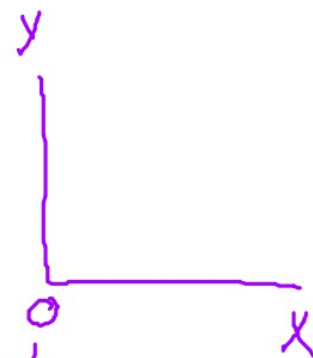
$$f(x) = 3^x$$

$$f(0) = 3^0 = 1$$

$$f(x) = \underline{a} \cdot b^x$$

$$f(x) = \underline{.9} * .5^x$$

$a = y\text{-intercept}$



$$f(x) = \underline{\underline{12}} * 0.01^x$$

## 3.4 Graphs of Exponential Functions

10/17/14

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

### 3.4 Graphs of Exponential Functions

10/17/14

Determine if an exponential function is an increasing or decreasing function when given its base.

$$f(x) = 0.9 * 3^x$$

$b > 1$  increasing

$$f(x) = 3 * \underline{.5}^x \quad \text{decreasing}$$

$0 < b < 1$

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

### 3.4 Graphs of Exponential Functions

10/17/14

Identify the domain, range, and y-intercept of exponential functions.

$$f(x) = 3 * 3^x$$

Domain: all Real numbers

Range:  $y > 0$

y-intercept: 3

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.



### 3.4 Graphs of Exponential Functions

10/17/14

Identify the domain, range, and y-intercept of exponential functions.

$$f(x) = 12 * 0.1^x$$

$$f(x) = \underline{a} b^{x \rightarrow 1}$$

$D$ : all Real numbers

$R$ :  $y > 0$

y-intercept: 12

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

### 3.4 Graphs of Exponential Functions

10/17/14

Identify the exponential function that represents a given graph.

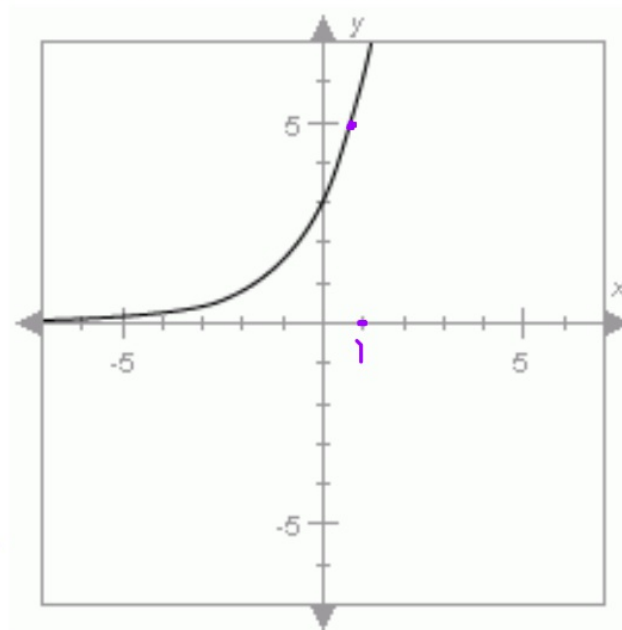
$$f(x) = a \cdot b^x \quad \text{growth}$$

$$a = y\text{-intercept} \quad b > 1$$

$$a = 3$$

$$f(x) = 3 \cdot \frac{5}{3}^x$$

$$\frac{5}{3} = \frac{b^1 \cdot 3}{3}$$
$$\frac{5}{3} = b$$



IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

### 3.4 Graphs of Exponential Functions

10/17/14

Identify the exponential function that represents a given graph.

$$f(x) = a b^x$$

$$a = 3$$

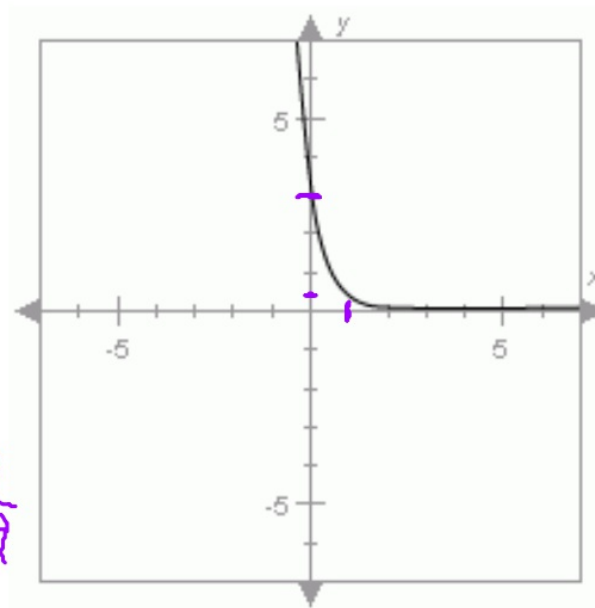
decay

$$0 < b < 1$$

$$f(x) = 3 \cdot \frac{1}{9}^x$$

$$\frac{3 b'}{3} = \frac{\frac{1}{3}}{3}$$

$$b = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$



IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.



## 3.4 Graphs of Exponential Functions

10/17/14

Vocabulary 3.4.1 p. 13

Practice 3.4.2

Apex quiz 3.4.3

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

### 3.4 Graphs of Exponential Functions

10/22/14

What facts are true for the graph of the functions below?

$$f(x) = \left(\frac{3}{5}\right)^x$$

$0 < b < 1$  decay  
decreasing

$y = a \cdot b^x$   
y-intercept:  $(0, 1)$

Domain: all Real numbers

Range:  $y > 0$

$$g(x) = 3 * 4^x$$

$b > 1$  growth  
increasing

y-intercept:  $(0, 3)$

Domain: all Real numbers

Range:  $y > 0$

Groups 1, 3, & 5 graph  $f(x)$ . Groups 2 & 4 graph  $g(x)$ .

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

### 3.4 Graphs of Exponential Functions

10/22/14

$$a = (0, 3)$$

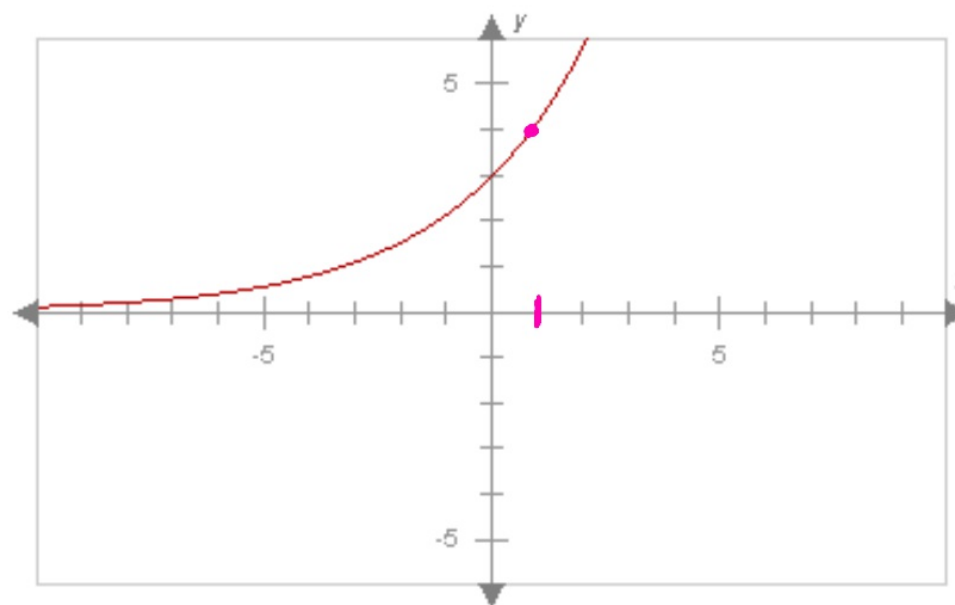
$$y = ab^x$$

$$\frac{4}{3} = \frac{3b^1}{3}$$

$$\frac{4}{3} = b$$

$$y = 3 \cdot \frac{4}{3}^x$$

The graph below could be the graph of which exponential function?



IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

### 3.4 Graphs of Exponential Functions

10/22/14

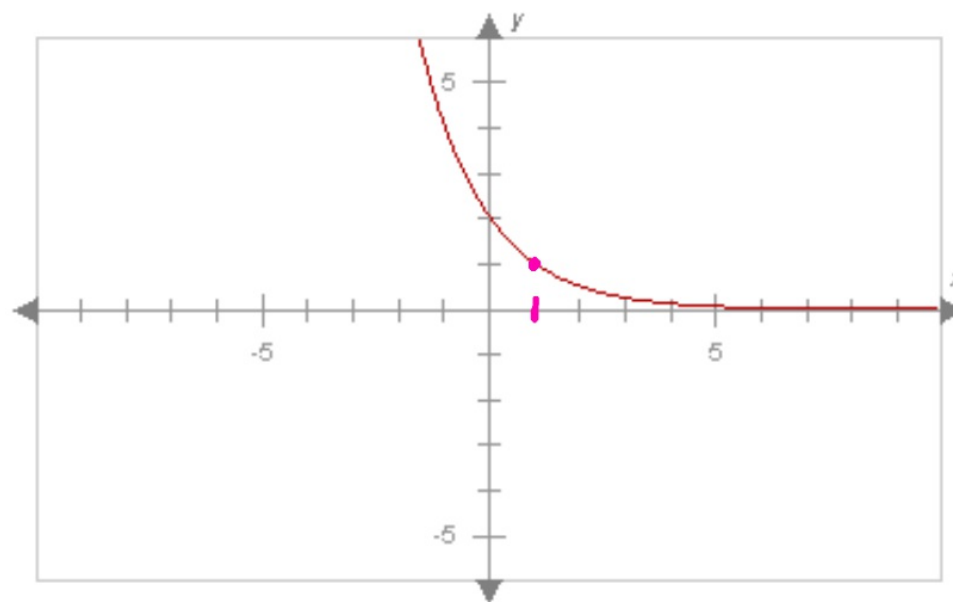
$$a=(0, 2), 0 < b < 1$$

$$\frac{1}{2} = \frac{2b}{2}$$

$$b = \frac{1}{2}$$

$$y = 2 \cdot \left(\frac{1}{2}\right)^x$$

The graph below could be the graph of which exponential function?



IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

## 3.4 Graphs of Exponential Functions

10/22/14

### Practice 3.4.2

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.



## 3.4 Graphs of Exponential Functions

10/22/14

Vocabulary 3.4.1 p. 13

Practice 3.4.2

Apex quiz 3.4.3

IWBAT determine if an exponential function is an increasing or decreasing function when given its base; identify the domain, range, and y-intercept of exponential functions; and identify the exponential function that represents a given graph.

### 3.5 Logarithmic Functions

10/23/14

Identify the domain, range, and y-intercept of exponential functions.

$$a^{-n} = \frac{1}{a^n}$$

$$f(x) = 4 * .8^x$$

$$y\text{-INT} = (0, 4)$$

$$\text{RANGE} = y > 0$$

$$\text{DOMAIN} = \text{ALL REAL NUMBERS}$$

$$\text{DECAY} = 0 < b < 1$$

$$g(x) = .01 * 99^x$$

$$y\text{-int} = .01$$

$$\text{Range} = y > 0$$

$$\text{Domain} = \text{All real numbers}$$

$$\text{growth} = b > 1$$

Groups 1, 3, & 5 graph  $f(x)$ . Groups 2 & 4 graph  $g(x)$ .

## 3.5 Logarithmic Functions

10/23/14

**The three properties of logarithms we will use today.**

Definition of a logarithm

$$a = b^c$$

$$\log_b a = c$$

Change of base property

$$\log_b a = \frac{\log a}{\log b}$$

Power property

$$\log(a^c) = c \log(a)$$

## 3.5 Logarithmic Functions

10/23/14

IWBAT convert exponential functions into common or natural logarithmic functions. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

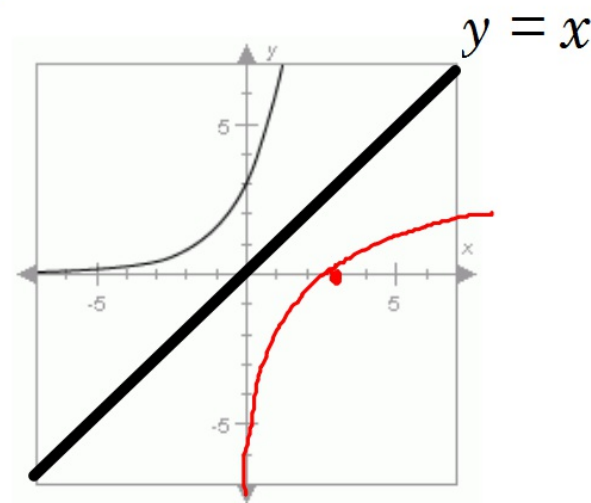
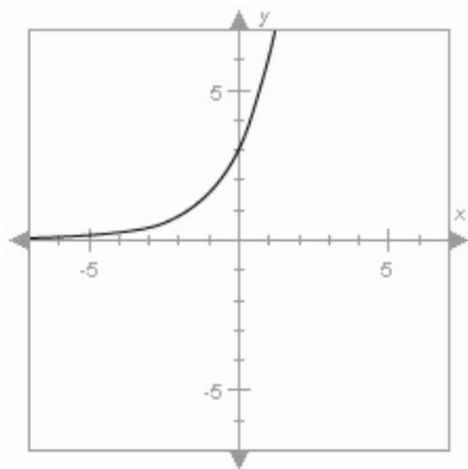
### 3.5 Logarithmic Functions

10/23/14

Logarithmic functions are the *inverse* of exponential functions.

Recall that inverses are flipped across the line  $y = x$ , and that all of the points of the original graph  $(x, y)$  become  $(y, x)$ .

Exponentials are functions. Are logarithms functions? What test can we run to tell us?



Convert common and natural logarithmic functions into exponential functions.



### 3.5 Logarithmic Functions

10/23/14

$$f(x) = b^x$$

$$f^{-1}(y) = \log_b y$$

$$a = b^c$$

$$c = \log_b a$$

$$\begin{aligned} \log(10^x) &= \log(100) \\ 10^x &= 100 \\ \Rightarrow x \frac{\log(10)}{\log(10)} &= \frac{\log(100)}{\log(10)} \\ x &= \frac{\log(100)}{\log(10)} = 2 \end{aligned}$$

$$\log_{10} 100 = x$$

$$\log_{10} 100 = 2$$

$$\log_3 243 = x$$

$$x = 5$$

$$\log_2 8 = x$$

$$x = 3$$

Convert common and natural logarithmic functions into exponential functions.

## 3.5 Logarithmic Functions

10/23/14

common logarithm

$$\log_{10} x = \log x$$

natural logarithm

$$y = \ln x$$

$$x = e^y$$

$$e \approx 2.718$$
$$\pi \approx 3.1415$$

Solve and rewrite in another form.

$$\log_{10} 1000 = x$$

$$10^x = 1000$$

$$\ln 1000 = x$$

$$e^x = 1000$$

Ch.B.

$$\log_2 8 = x$$
$$\frac{\log 8}{\log 2} = x$$

Convert common and natural logarithmic functions into exponential functions.

## 3.5 Logarithmic Functions

10/23/14

Definition of a logarithm

$$a = b^c$$

$$\log_b a = c$$

Change of base property

$$\log_b a = \frac{\log a}{\log b}$$

Power property

$$\log(a^c) = c \log(a)$$

Vocabulary 3.5.1 p. 10

Practice 3.5.2

Apex quiz 3.5.3

Convert common and natural logarithmic functions into exponential functions.

### 3.5 Logarithmic Functions

10/27/14

Convert exponential functions into common or natural logarithmic functions.

$$a) 10^x = 85$$

$$\log_{10} 85 = x$$

$$\log 85 = x$$

$$b) 10^x = \frac{1}{100}$$

$$\log_{10} \left( \frac{1}{100} \right) = x$$

$$\log \left( \frac{1}{100} \right) = x$$

$$c) 2^x = 73$$

$$\log_2 73 = x$$

$$\frac{\log 73}{\log 2} = x$$

$$d) e^x = 12$$

$$\ln 12 = x$$

Convert common and natural logarithmic functions into exponential functions.

## 3.5 Logarithmic Functions

10/27/14

### Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



### 3.5 Logarithmic Functions

10/27/14

What logarithmic equation is equivalent to the exponential equation below?

a)  $4^c = 256$       $\log_4 256 = c$

b)  $e^a = 55$       $\ln 55 = a$

c)  $5^c = 250$       $\log_5 250 = c$

True or False:

A logarithmic function is the inverse of an exponential function.

Convert common and natural logarithmic functions into exponential functions.

### 3.5 Logarithmic Functions

10/27/14

What exponential equation is equivalent to the logarithmic equation below?

a)  $c = \ln 3$       $e^c = 3$

b)  $\log 300 = a$       $10^a = 300$

c)  $\log 987 = a$       $10^a = 987$

What function is the inverse of  $f(x) = b^x$ ?

$$f^{-1}(y) = \log_b y$$

Convert common and natural logarithmic functions into exponential functions.

## 3.5 Logarithmic Functions

10/27/14

### Practice 3.5.2

$$2) \log z = 23$$

$$10^{23} = z$$

Convert common and natural logarithmic functions into exponential functions.

## 3.5 Logarithmic Functions

10/27/14

Vocabulary 3.5.1 p. 10

Practice 3.5.2

Apex quiz 3.5.3

Convert common and natural logarithmic functions into exponential functions.

### 3.6 Graphs of Logarithmic Functions

10/28/14

Convert exponential functions into common or natural logarithmic functions.

$$f(x) = b^x \qquad f^{-1}(y) = \log_b y$$

$$24 = 3^{3x} \quad \log_3 24 = 3x$$

$$181 = e^x$$
$$\ln 181 = x$$

$$63 = 5^x \quad \log_5 63 = x$$



### 3.6 Graphs of Logarithmic Functions

10/28/14

Evaluate the natural log function for a given input value.

Find  $f(5)$ .

$$\ln(3x) = \ln(3 \cdot 5) = \ln(15) \approx 2.70$$

$$\ln\left(\frac{x}{5}\right) = \ln(1) = 0 \quad \text{X-intercept } (1, 0)$$

$$\ln\left(\frac{3x}{5}\right) = \ln(3) \approx 1.09$$

## 3.6 Graphs of Logarithmic Functions

10/28/14

IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range, x-intercept, and asymptote, and decide whether it decreases or increases given a logarithmic function. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

### 3.6 Graphs of Logarithmic Functions

10/28/14

Determine the values for which a logarithmic function increases or decreases.

$$\log_2 x = \frac{\log x}{\log 2}$$

increases  
 $x > 0$

$$\ln_2 x = \frac{\ln(x)}{\ln(2)}$$

increasing  
 $x > 0$

$$\log_8 x = \frac{\log(x)}{\log(8)}$$

increasing  
 $x > 0$

$$\ln_8 x$$

$b > 1$   
increasing

$$\log_{0.5} x = \frac{\log(x)}{\log(0.5)}$$

Decreasing

$$\ln_{0.5} x$$

$0 < b < 1$   
decreasing

Domain:  $x > 0$

Range: all Real #

IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range, x-intercept, and asymptote, and decide whether it decreases or increases given a logarithmic function.

### 3.6 Graphs of Logarithmic Functions

10/28/14

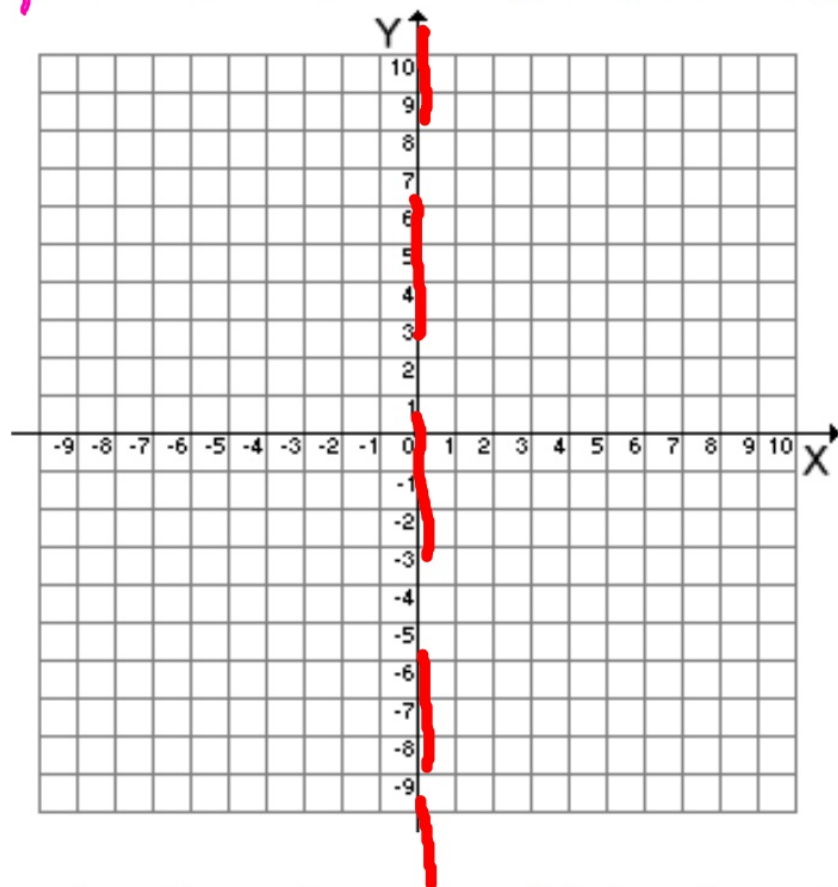
Identify the domain, range, x-intercept, and asymptote.

Domain:  $x > 0$   $\log_2 x$

Range: all Real #

x-intercept:  $(1, 0)$

Vertical asymptote:  $x = 0$



IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range, x-intercept, and asymptote.



### 3.6 Graphs of Logarithmic Functions

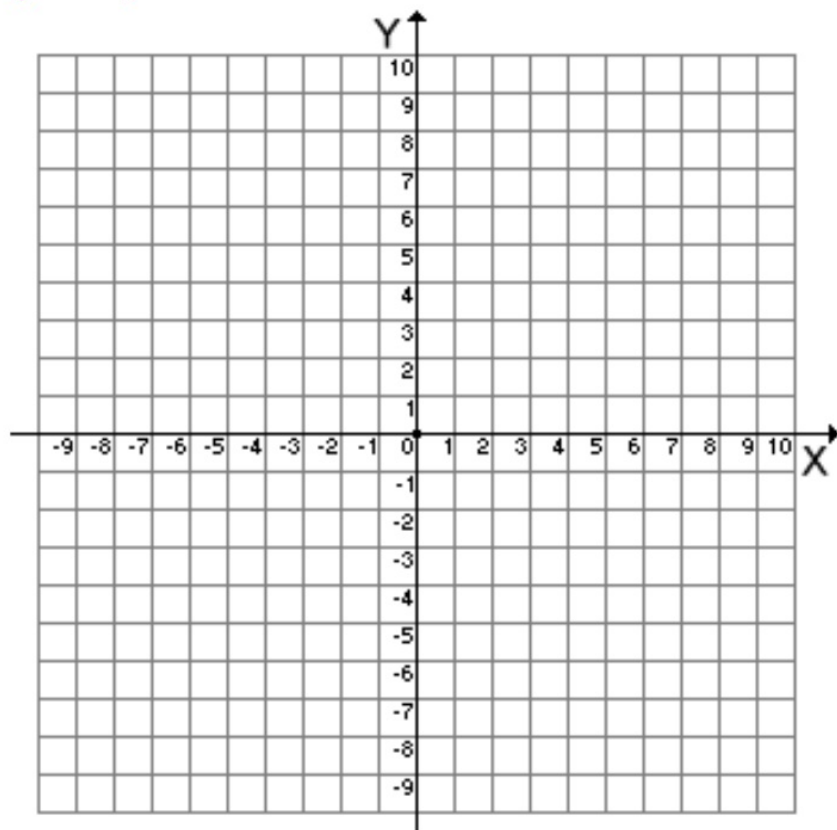
10/28/14

Identify the domain, range, x-intercept, and asymptote.

$$D: x > 0$$
$$x\text{-i}: (1, 0)$$

$$\ln_{0.5} x$$

$$R: \text{all Real \#}$$
$$a: x = 0$$



IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range, x-intercept, and asymptote.



## 3.6 Graphs of Logarithmic Functions

10/28/14

Vocabulary 3.6.1 p. 15

Practice 3.6.2

Apex quiz 3.6.3

IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range, x-intercept, and asymptote.

### 3.6 Graphs of Logarithmic Functions

10/29/14

**Convert common or natural logarithmic functions into exponential functions.**

$$f(x) = \log_b x$$

$$f^{-1}(y) = b^y$$

$$\ln(181) = x$$

$$e^x = 181$$

$$\log_3 154 = x$$

$$3^x = 154$$

$$\log_{0.5} x = 26$$

$$0.5^{26} = x$$

IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range, x-intercept, and asymptote.

## 3.6 Graphs of Logarithmic Functions

10/29/14

### Practice 3.6.2

$D: x > 0$      $R: \text{all real numbers}$

$x$ -intercept:  $(1, 0)$

increasing:  $b > 1$

decreasing:  $0 < b < 1$

IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range,  $x$ -intercept, and asymptote.

## 3.6 Graphs of Logarithmic Functions

10/29/14

Vocabulary 3.6.1 p. 15

Practice 3.6.2

Apex quiz 3.6.3

IWBAT determine the values for which a logarithmic function increases or decreases, and identify the domain, range, x-intercept, and asymptote.

### 3.7 Properties of Exponents & Logarithms

10/30/14

**Evaluate the natural log function for a given input value. Give your answer to three decimal places.**

$$f(x) = \ln(3x)$$

$$f(3) = 2.197$$

$$g(x) = 2\ln(x)$$

$$g(1) = 0$$

$$h(x) = \ln\left(\frac{4x}{3}\right)$$

$$h(6) = \ln(8) = 2.079$$

$$j(x) = 2 + 3\ln\left(\frac{4}{x}\right)$$

$$j(7) = 2 + 3\ln\left(\frac{4}{7}\right) = -3.21$$



### 3.7 Properties of Exponents & Logarithms

10/30/14

**Apply the properties of exponents and logarithms to simplify expressions.**

Name this property and rewrite the two examples.

$$\log_b(a) = \frac{\log_c(a)}{\log_c(b)} \quad \text{Change of base property}$$

$$\log_2(x) = \frac{\log(x)}{\log(2)}$$

$$\log_5(12) = \frac{\log(12)}{\log(5)}$$

## 3.7 Properties of Exponents & Logarithms

10/30/14

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

### 3.7 Properties of Exponents & Logarithms

10/30/14

#### Multiplication Law of Exponents & Logarithms

$$\log(a * b) = \log(a) + \log(b)$$

$$\log(15) = \log(3 \cdot 5) = \log(3) + \log(5)$$

$$\log(6) + \log(5) = \log(6 \cdot 5) = \log(30)$$

$$a^m * a^n = a^{m+n}$$

$$8^6 * 8^5 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 8^{11}$$

$$8^{\frac{1}{2}} * 8^{\frac{1}{4}} = 8^{\frac{3}{4}}$$

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.

### 3.7 Properties of Exponents & Logarithms

10/30/14

#### Subtraction Law of Exponents & Logarithms

$$\log(a) - \log(b) = \log\left(\frac{a}{b}\right)$$

$$\log(\underset{3}{6}) - \log(\underset{4}{8}) = \log\left(\frac{6}{8}\right) = \log\left(\frac{3}{4}\right)$$

$$\log\left(\frac{11}{17}\right) = \log(11) - \log(17)$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{2^3}{2^7} = 2^{3-7} = 2^{-4} = \frac{1}{2^4}$$

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2$$

Negative exponent property

$$a^{-n} = \frac{1}{a^n}$$

$$2^3 = \frac{1}{2^{-3}}$$

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.

### 3.7 Properties of Exponents & Logarithms

10/30/14

#### Power Property of Logarithms

$$\log(a^m) = m\log(a)$$

$$\log(7^x) = x \log(7)$$

$$\log\left(\frac{1}{3}^{2x}\right) = 2x \log\left(\frac{1}{3}\right) = 2x[\log(1) - \log(3)]$$

[ { [ (

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.



### 3.7 Properties of Exponents & Logarithms

10/30/14

These are always true if  $b > 0$  and  $b \neq 1$ .

$$\log_b 1 = 0$$

$$b^0 = 1$$

$$\log_b b = 1$$

$$b^1 = b$$

$$(a^m)^n = a^{m \cdot n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.

## 3.7 Properties of Exponents & Logarithms

10/30/14

Vocabulary 3.7.1 p. 21

Practice 3.7.2

Apex quizzes 3.7.3, 3.7.4, & 3.7.5

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.

### 3.7 Properties of Exponents & Logarithms

10/31/14

**Apply the properties of exponents and logarithms to change expressions to an equivalent form.**

$$a^m \cdot a^n = a^{m+n}$$

$$\log(3) + \log(7) = \log(3 \cdot 7) = \log(21)$$

$$5\log(7) = \log(7^5)$$

$$\log(5) - \log(2) = \log\left(\frac{5}{2}\right) = \log(2.5)$$

$$\left(\frac{3}{7}\right)^x = \frac{3^x}{7^x}$$

$$\left(\frac{3}{7}\right)^2 = \frac{3}{7} \cdot \frac{3}{7} = \frac{9}{49} = \frac{3^2}{7^2}$$

$$64^x = 8^x \cdot 8^x = 8^{2x}$$

$$4^x \cdot 16^x = 4^x \cdot 4^x \cdot 4^x = 4^{3x}$$

$$2^{2x} \cdot 2^{2x} \cdot 2^{2x} = 2^{6x}$$

$$\frac{64^x}{4^x} = \left(\frac{64}{4}\right)^x = \left(\frac{32}{1}\right)^x = 32^x$$

$$\frac{4^{3x}}{4^x} = 4^{2x}$$

Black: one equivalent form is acceptable

Blue: as many equivalent forms as you can think of

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.

## 3.7 Properties of Exponents & Logarithms

10/31/14

### Practice 3.7.2

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.

## 3.7 Properties of Exponents & Logarithms

10/31/14

Vocabulary 3.7.1 p. 21

Practice 3.7.2

Apex quizzes 3.7.3, 3.7.4, & 3.7.5

IWBAT identify an equivalent logarithmic expression using the change-of-base formula, and evaluate logarithmic expressions.



### 3.8 Solving Exponential Equations

11/03/14

Apply the properties of exponents and logarithms to simplify expressions.

Simplify without using a calculator.

$$\log(7) + \log(6) \quad \log(7 \cdot 6) = \log(42)$$

$$\log(12) - \log(9) \quad \log\left(\frac{12}{9}\right) \quad \log\left(\frac{4}{3}\right)$$

Are these equivalent? If not, change the right-hand side to make them equivalent.

$$4^5 * 4^7 = 4^{35} \quad 4^{12}$$

$$\log_4(12) = \cancel{\log(4) * \log(12)} \\ \log(12) \div \log(4)$$

## 3.8 Solving Exponential Equations

11/03/14

IWBAT use properties of exponents and logarithms to solve exponential equations. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

### 3.8 Solving Exponential Equations

11/03/14

Use properties of exponents and logarithms to solve exponential equations.

$$\begin{aligned} e^x &= 12 \\ \ln(e^x) &= \ln(12) \\ x \ln(e) &= \ln(12) \\ x &\approx 2.485 \end{aligned}$$

$$\begin{aligned} 27^x &= 155 \\ \log(27^x) &= \log(155) \\ x \log(27) &= \log(155) \\ \frac{x \log(27)}{\log(27)} &= \frac{\log(155)}{\log(27)} \\ x &\approx 1.53 \end{aligned}$$

$$10^x = 35$$

$$\begin{aligned} x &= \log(35) \\ x &\approx 1.54 \end{aligned}$$

$$e^x = 178$$

$$\begin{aligned} \ln(e^x) &= \ln(178) \\ x \ln(e) &= \ln(178) \\ x &= \ln(178) \\ x &\approx 5.182 \end{aligned}$$

IWBAT use properties of exponents and logarithms to solve exponential equations.

### 3.8 Solving Exponential Equations

11/03/14

Use properties of exponents and logarithms to solve exponential equations.

$$\frac{4 * 3^x}{4} = \frac{2.62}{4}$$

$$3^x = 0.655$$

$$\log(3^x) = \log(0.655)$$

$$\frac{x \log(3)}{\log(3)} = \frac{\log(0.655)}{\log(3)}$$

$$x \approx -0.385$$

$$e^{3x} = 162754.79$$

$$\frac{3x \ln(e)}{3} = \frac{\ln(162754.79)}{3}$$

$$x \approx 4$$

$$\frac{6 * e^x}{6} = \frac{12.36}{6}$$

$$e^x = 2.06$$

$$\ln(e^x) = \ln(2.06) \quad x \approx 0.722$$

$$\ln(e) = \ln(2.06)$$

$$2^{5x} = 1024$$

$$\log(2^{5x}) = \log(1024)$$

$$\frac{5x \log(2)}{5} = \frac{\log(1024)}{5}$$

$$\frac{x \log(2)}{\log(2)} = \frac{(5 \log(2))}{\log(2)}$$

$$x = 2$$

IWBAT use properties of exponents and logarithms to solve exponential equations.



## 3.8 Solving Exponential Equations

11/03/14

Vocabulary 3.8.1 p. 15

Practice 3.8.2

Apex quizzes 3.8.3

IWBAT use properties of exponents and logarithms to solve exponential equations.



### 3.9 Solving Logarithmic Equations

11/04/14

Rewrite a logarithmic equation into an equivalent exponential equation.

$$e^x = 12$$
$$\times \ln(e) = \ln(12)$$
$$x \approx 2.48$$

$$10^x = 35$$
$$\times \frac{\log(10)}{\log(10)} = \frac{\log(35)}{\log(10)}$$
$$x \approx 1.544$$

$$e^x = 178$$
$$\ln(e^x) = \ln(178)$$
$$\times \ln(e) = \ln(178)$$
$$x = \ln(178)$$
$$x \approx 5.182$$

$$27^x = 155$$
$$\times \frac{\log(27)}{\log(27)} = \frac{\log(155)}{\log(27)}$$
$$x \approx 1.53$$

### 3.9 Solving Logarithmic Equations

11/04/14

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

### 3.9 Solving Logarithmic Equations

11/04/14

Use properties of exponents and logarithms to solve exponential equations.

$$a^x = c$$

$$\log_a(c) = x$$

$$\log\left(\frac{x}{17}\right) = 5$$

$$\log(x) - \log(17) = 5$$

$$+ \log(17) \quad + \log(17)$$

$$\log(x) = 5 + \log(17)$$

$$x = 10^{(5 + \log(17))}$$

$$x = 1,700,000$$

$$\log(17x) = 5$$

$$\frac{17x}{17} = \frac{10^5}{17}$$

$$x = \frac{10^5}{17}$$

$$x = 5882.35$$

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation.

### 3.9 Solving Logarithmic Equations

11/04/14

Use properties of exponents and logarithms to solve exponential equations.

$$\log\left(\frac{x}{6}\right) = 7$$

$$10^{(7 + \log(6))}$$
$$60,000,000$$

$$\log\left(\frac{x}{6}\right) = 7$$
$$10^{-1} \quad 10^7$$

$$6 \cdot \frac{x}{6} = 10^7 \cdot 6$$

$$x = 6 \times 10^7$$

$$\log(6x) = 2.5$$

$$\frac{6x}{6} = \frac{10^{2.5}}{6}$$
$$x = 52.70$$

$$\log(6) + \log(x) = 2.5$$

$$\log(x) = 2.5 - \log(6)$$
$$10^{-2} \quad 10^1$$

$$x = 52.7$$

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation.



### 3.9 Solving Logarithmic Equations

11/04/14

Rewrite an equation that includes a natural log into an equivalent exponential equation.

$$\ln(5) = x$$

$e^{\cancel{1}} \quad e^{\cancel{1}}$

$$5 = e^x$$

$$\cancel{\frac{5}{1}} \cdot \cancel{\frac{1}{5}} \ln(6) = x \cdot \frac{5}{1}$$

$\cancel{\ln(6)}^{\cancel{1}} = \cancel{5}x$

$$6 = e^{5x}$$

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation.



## 3.9 Solving Logarithmic Equations

11/04/14

Vocabulary 3.9.1 p. 15

Practice 3.9.2

Apex quizzes 3.9.3

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation.

### 3.9 Solving Logarithmic Equations

11/05/14

**Rewrite a logarithmic equation into an equivalent exponential equation. Do not solve for  $x$ .**

$$x = \log_{0.4}(5)$$

$$0.4^x = 5$$

$$x = \ln(0.9)$$

$$e^x = 0.9$$

$$\frac{15}{2} = \frac{2}{2} \log_2(2x)$$

$$7.5 = \log_2(2x)$$

$$2^{7.5} = 2x \text{ power eq.}$$

$$x = \frac{1}{3} \log_{0.8}(0.3)$$

$$3x = \log_{0.8}(0.3)$$

$$0.8^{3x} = 0.3$$

$$x = \frac{1}{5} \ln(12)$$

$$5x = \ln(12)$$

$$e^{5x} = 12$$

$$3 = \frac{2}{3} \ln\left(\frac{x}{2}\right)$$

power eq.

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation.

## 3.9 Solving Logarithmic Equations

11/05/14

### Practice 3.9.2

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation.

## 3.9 Solving Logarithmic Equations

11/05/14

Vocabulary 3.9.1 p. 15

Practice 3.9.2

Apex quiz 3.9.3

IWBAT use properties of exponents and logarithms to solve logarithmic equations, and rewrite an equation that includes a natural log into an equivalent exponential equation.

### 3.10 Exponents, Logarithms, and Their Graphs

11/06/14

Use properties of exponents and logarithms to solve logarithmic equations.

Solve for  $x$ .

$$\frac{25}{25} \log\left(\frac{17}{x}\right) = \frac{12.5}{25}$$

$$10^{-1} \log\left(\frac{17}{x}\right) = \frac{25}{10^1}$$

$$\frac{1}{17} \cdot \frac{17}{x} = 10^{.5} \cdot \frac{1}{17}$$

$$\frac{1}{x} = \frac{10^{.5}}{17}$$

$$x = \frac{17}{10^{.5}}$$

$$x = 5.37$$

$$\frac{\frac{2}{3}}{\frac{2}{3}} \log(2x) = \frac{1.987}{\frac{2}{3}}$$

$$10^{-1} \log(2x) = \frac{3(1.987)}{2}$$

$$\frac{2x}{2} = \frac{10^{\frac{3(1.987)}{2}}}{2}$$

$$x = 478$$



IWBAT use logarithms to solve exponential decay problems and exponential growth problems. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

#### Cooling coffee problem.

You pour a cup of coffee and leave it on the kitchen table. How long do you have to wait before you can drink it without burning your mouth?

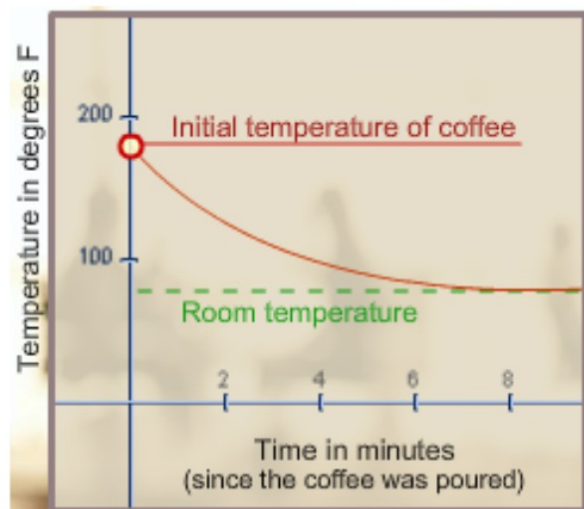
Which of these choices do you think best describes how a cup of coffee cools?

- ☐ Drops slowly and steadily to room temperature
- ☒ ~~Drops quickly to below room temperature and then bounces back~~
- ☐ Drops quickly but then levels off as it gets closer to room temperature

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

### 3.10 Exponents, Logarithms, and Their Graphs

11/06/14



This is an example of what type of function?

exponential decay

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

### 3.10 Exponents, Logarithms, and Their Graphs

11/06/14

Newton's Law of Cooling

$$T(t) = T_A + (T_0 - T_A)e^{-kt}$$

$T_A$ : ambient temperature

$T_0$ : initial temperature of the object

$t$ : elapsed time

$k$ : positive constant dependent on the situation

If we know that  $T_0 = 200$ ,  $T_A = 68$ , and  $T(10) = 150$ ,  
when will the coffee reach 100 degrees Fahrenheit?

find  $k$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.



### 3.10 Exponents, Logarithms, and Their Graphs

11/06/14

Newton's Law of Cooling

$$T(t) = T_A + (T_0 - T_A)e^{-kt}$$

If we know that  $T_0 = 200$ ,  $T_A = 68$ , and  $T(10) = 150$ , when will the coffee reach 100 degrees Fahrenheit?

$$150 = 68 + (200 - 68)e^{-k(10)}$$

$$\underset{-68}{150} = \underset{-68}{68} + 132e^{-k(10)}$$

$$\frac{82}{132} = \frac{132e^{-10k}}{132}$$

$$\ln(0.621) = \ln(e^{-10k})$$

$$\frac{\ln\left(\frac{41}{66}\right)}{-10} = \frac{-10k}{-10}$$

$$k = \frac{\ln\left(\frac{41}{66}\right)}{-10} = 0.0476$$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.



### 3.10 Exponents, Logarithms, and Their Graphs

11/06/14

Newton's Law of Cooling

$$T(t) = T_A + (T_0 - T_A)e^{-kt}$$

If we know that  $T_0 = 200$ ,  $T_A = 68$ , and  $T(10) = 150$ ,  
when will the coffee reach 100 degrees Fahrenheit?

$$k = 0.0478$$

$$100 = 68 + (200 - 68)e^{-0.0478t}$$

$$\frac{32}{132} = \frac{132}{132}e^{-0.0478t}$$

$$\ln\left(\frac{8}{33}\right) = \ln\left(e^{-0.0478t}\right)$$

$$\frac{\ln\left(\frac{8}{33}\right)}{-0.0478} = \frac{-0.0478t}{-0.0478}$$

$$t = \frac{\ln\left(\frac{8}{33}\right)}{-0.0478} = 29.8 \text{ min}$$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

### 3.10 Exponents, Logarithms, and Their Graphs

11/06/14

Carbon dating

$$C(t) = C_0 \left( 2^{\frac{-t}{5730}} \right)$$

If an artifact contains 2.1% of the original  $C^{14}$ , approximately how old is the artifact? *find t*

$$\frac{0.021 C_0}{C_0} = \frac{C_0 \left( 2^{\frac{-t}{5730}} \right)}{C_0}$$

$$\log(0.021) = \log \left( 2^{\frac{-t}{5730}} \right)$$

$$t = - \frac{5730 \log(0.021)}{\log(2)}$$

$$5730 \cdot \log(0.021) = \frac{-t}{5730} \log(2) \cdot 5730 \quad t \approx 31,935 \text{ yr.}$$

$$\frac{5730 \log(0.021)}{-\log(2)} = \frac{-t \log(2)}{+\log(2)}$$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

### 3.10 Exponents, Logarithms, and Their Graphs

11/07/14

Use properties of exponents and logarithms to solve logarithmic equations.

Solve for  $x$ .

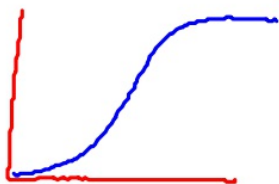
$$\begin{aligned} 27(0.93^x) &= 12 \\ \div 27 & \quad \div 27 \\ x \log(0.93) &= \frac{4}{0.93} \\ x &= 12.63 \end{aligned}$$

$$\begin{aligned} 23 + 45(1.024^x) &= 147 \\ -23 & \quad -23 \\ 45(1.024^x) &= \frac{124}{45} \\ \log & \quad \log \\ (1.024^x) &= 2.75 \\ \log_{1.024} 2.75 &= x \\ x \log(1.024) &= \frac{\log(2.75)}{\log(1.024)} \\ x &= 42.65 \end{aligned}$$

### 3.10 Exponents, Logarithms, and Their Graphs

11/07/14

#### Carrying Capacity (Logistic Growth)



$$P(t) = \frac{1000}{1+9e^{-0.2t}}$$

$t$ : time since the introduction of the species to the area

How many animals were in the initial group?

$$t=0 \quad P(0) = \frac{1000}{1+9e^{-0.2(0)}} = \frac{1000}{1+9} = \frac{1000}{10} = 100$$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.



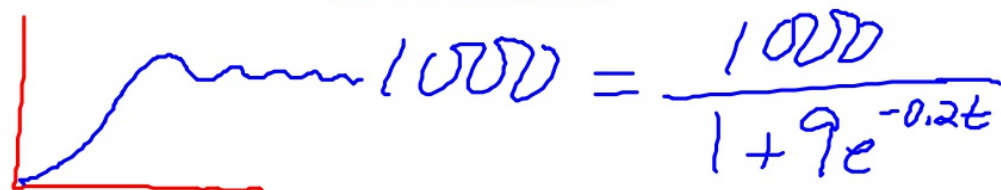
### 3.10 Exponents, Logarithms, and Their Graphs

11/07/14

#### Carrying Capacity (Logistic Growth)

$$P(t) = \frac{1000}{1 + 9e^{-0.2t}} \quad t: \text{time since the introduction of the species to the area}$$

How long until the full capacity of 1000 animals is reached?



$$1000 = \frac{1000}{1 + 9e^{-0.2t}}$$

$$1 + 9e^{-0.2t} = 1$$

$$\frac{9e^{-0.2t}}{9} = \frac{0}{9}$$

$$e^{-0.2t} = 0$$

$$-0.2t \ln(e) = \ln(0)$$

$$999 = \frac{1000}{1 + 9e^{-0.2t}}$$

$$1 + 9e^{-0.2t} = \frac{1000}{999} \approx 1.001$$

$$\frac{9e^{-0.2t}}{9} \approx \frac{0.001}{9}$$

$$\ln(e^{-0.2t}) \approx \ln\left(\frac{0.001}{9}\right)$$

$$\frac{-0.2t \ln(e)}{-0.2} \approx \frac{\ln\left(\frac{0.001}{9}\right)}{-0.2}$$

$$t \approx 45.52 \text{ yr}$$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.



## 3.10 Exponents, Logarithms, and Their Graphs

11/07/14

Vocabulary 3.10.1 p. 16

Practice 3.10.2

Apex quiz 3.10.3

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

### 3.10 Exponents, Logarithms, and Their Graphs

11/12/14

**Use properties of exponents and logarithms to solve exponential equations.**

$$f(x) = \frac{12000}{1 + 499 \cdot 1.09^{-x}}$$

The equation  $f(x)$  gives the total sales  $x$  days after the release of a new video game.

Find  $x$  when  $f(x) = 6000$ .

$$\begin{aligned} 6000 &= \frac{12000}{1 + 499 \cdot 1.09^{-x}} \\ 1 + 499 \cdot 1.09^{-x} &= \frac{12000}{6000} \\ 1 + 499 \cdot 1.09^{-x} &= 2 \\ -1 & \quad -1 \\ 499 \cdot 1.09^{-x} &= 1 \\ \frac{499}{499} & \quad \frac{499}{499} \\ 1.09^{-x} &= \frac{1}{499} \end{aligned}$$

$$\begin{aligned} \log(1.09^{-x}) &= \log\left(\frac{1}{499}\right) \\ -x \log(1.09) &= \log\left(\frac{1}{499}\right) \\ \frac{-x \log(1.09)}{\log(1.09)} &= \frac{\log\left(\frac{1}{499}\right)}{\log(1.09)} \\ -1 \cdot -x &= \frac{\log\left(\frac{1}{499}\right)}{\log(1.09)} \cdot -1 \\ x &\approx 72 \text{ days} \end{aligned}$$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

### 3.10 Exponents, Logarithms, and Their Graphs

11/12/14

#### Practice 3.10.2

$$1) 60 =$$

$$.60 =$$

$$\frac{206}{2.07}$$

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

## 3.10 Exponents, Logarithms, and Their Graphs

11/12/14

### Practice 3.10.2

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.

## 3.10 Exponents, Logarithms, and Their Graphs

11/12/14

Vocabulary 3.10.1 p. 16

Practice 3.10.2

Apex quiz 3.10.3

IWBAT use logarithms to solve exponential decay problems and exponential growth problems.



### 3.11 Comparing and Analyzing Function Types

11/14/14

**Use properties of exponents and logarithms to solve logarithmic equations.**

$$D = 10 \log\left(\frac{I}{10^{-16}}\right)$$

This formula gives the loudness of a sound,  $D$ , measured in decibels (dB) where  $I$  is the intensity measured in Watts per square cm ( $\text{W}/\text{cm}^2$ ) and  $10^{-16} \text{ W}/\text{cm}^2$  is the approximate intensity of the least sound audible to the human ear.

Find the intensity of the sound experienced by the orchestra members seated in front of the brass section, measured at 107 dB.

$$\frac{107}{10} = \frac{10}{10} \log\left(\frac{I}{10^{-16}}\right)$$

$$I = 10^{(10.7 + \log(10^{-16}))}$$

$$10.7 = \log\left(\frac{I}{10^{-16}}\right)$$

$$I = 5.01 \times 10^{-6}$$

$$0.000005.01$$

$$10.7 = \log(I) - \log(10^{-16})$$

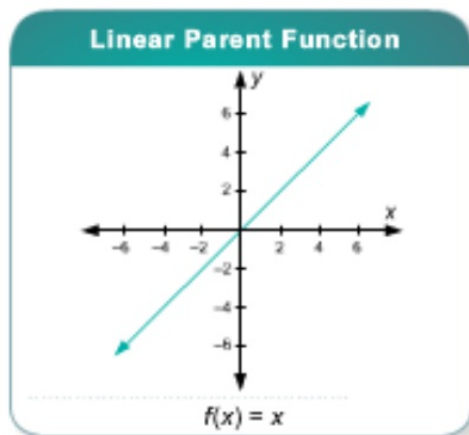
$$(10.7 + \log(10^{-16})) = \log(I)$$

$$10^1$$

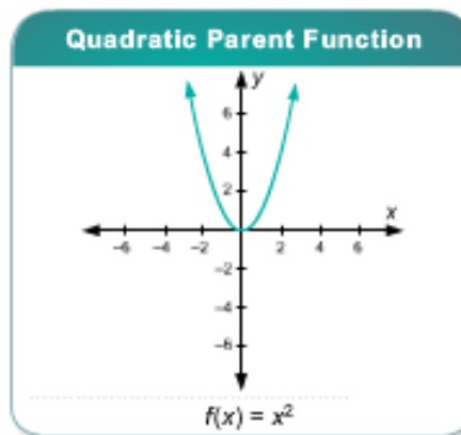
IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

## 3.11 Comparing and Analyzing Function Types Function Families

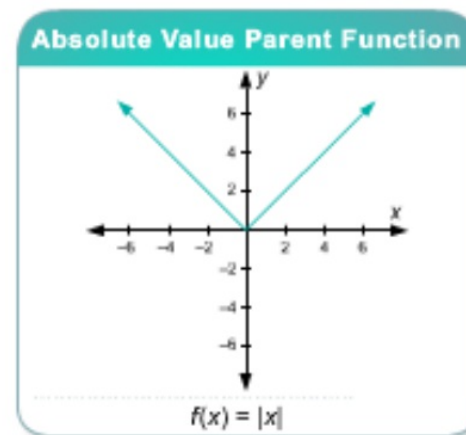
11/14/14



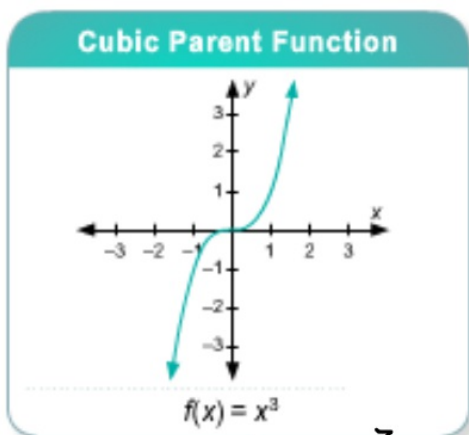
$$f(x) = x$$



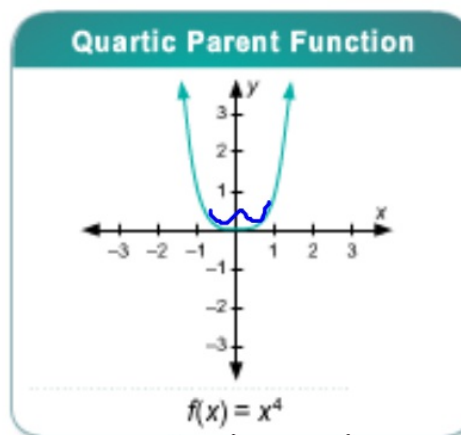
$$f(x) = x^2$$



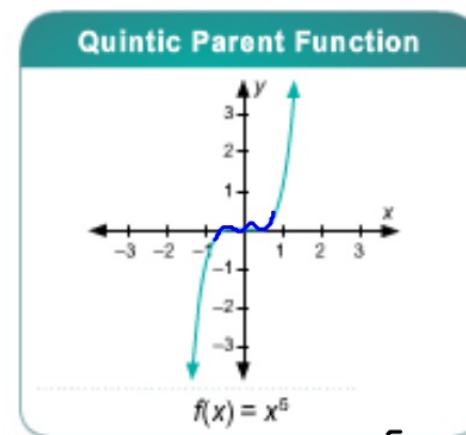
$$f(x) = |x|$$



$$f(x) = x^3$$



$$f(x) = x^4$$



$$f(x) = x^5$$

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

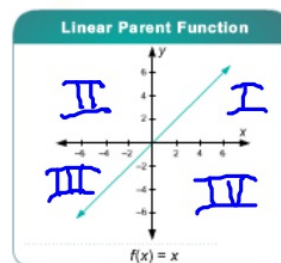
### 3.11 Comparing and Analyzing Function Types

11/14/14

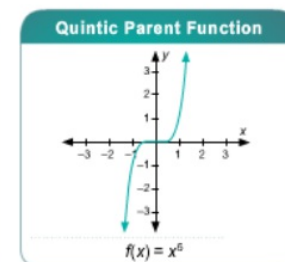
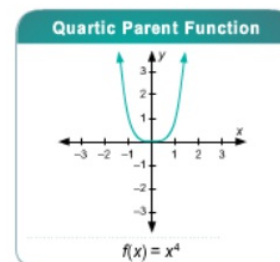
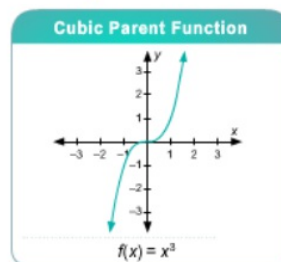
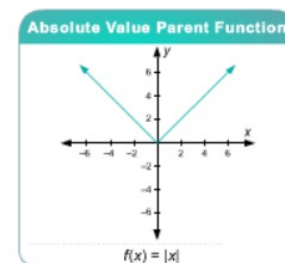
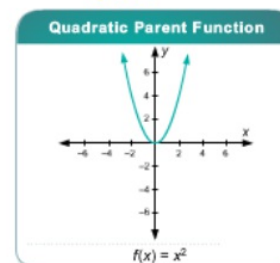
What do you notice about these functions and the degree of the polynomial?

Odd exponent:  
always in quad. I + III

Even exponent:  
always in quad. I + II



$$f(x) = x$$



IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



### 3.11 Comparing and Analyzing Function Types

11/14/14

**What qualifies as a function transformation?**

Transformation
Reflection over the line $y = x$
Reflection over the $x$ -axis
Reflection over the $y$ -axis
<sup>1</sup> Horizontal shift, slide, or translation
<sup>2</sup> Vertical shift, slide, or translation
Vertical stretch
Vertical compression

horizontal compression

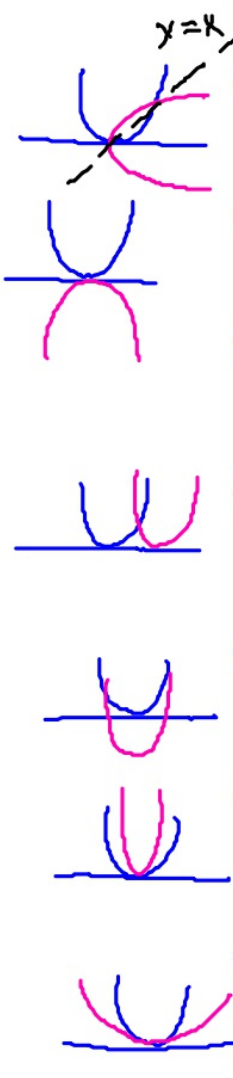
horizontal stretch

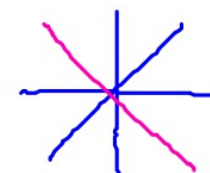
IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



## 3.11 Comparing and Analyzing Function Types

11/14/14

	Transformation	Transforming the function	Transforming the table values	Transforming the graph
	Reflection over the line $y = x$	Switch the variables $x$ and $y$ in the equation.	Switch the $x$ - and $y$ -values in the table.	Switch the $x$ - and $y$ -values in the graph.
	Reflection over the $x$ -axis	Multiply the function by <u><math>-1</math></u> : $-f(x)$ .	Multiply all $y$ -values in the table by $-1$ .	Flip the graph horizontally over the $x$ -axis.
	Reflection over the $y$ -axis	Substitute <u><math>-x</math></u> for $x$ and simplify: $f(-x)$ .	Multiply all $x$ -values in the table by $-1$ .	Flip the graph vertically over the $y$ -axis.
	<sup>1</sup> Horizontal shift, slide, or translation	Substitute <u><math>x - k</math></u> for $x$ and simplify: $f(x - k)$ .	Subtract a constant $k$ from the $x$ -values in the table.	Move the graph $k$ units horizontally.
	<sup>2</sup> Vertical shift, slide, or translation	Add a constant $k$ to the function itself: $f(x) + k$ .	Add a constant $k$ to the $y$ -values in the table.	Move the graph $k$ units vertically.
	Vertical stretch	Multiply the function by a <u>value greater than 1</u> : $k \cdot f(x)$ .	Multiply all $y$ -values by a value greater than 1.	Stretch the graph along the $y$ -axis.
	Vertical compression	Multiply the function by a <u>value between 0 and 1</u> : $k \cdot f(x)$ .	Multiply all $y$ -values by a value between 0 and 1.	Compress the graph along the $y$ -axis.

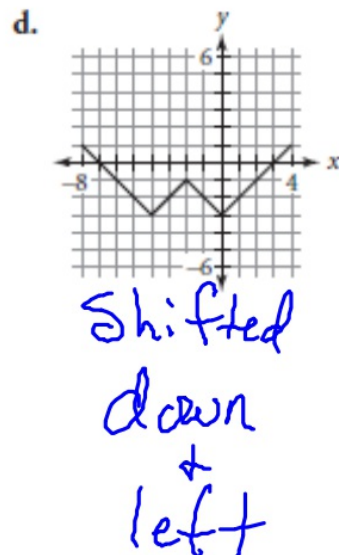
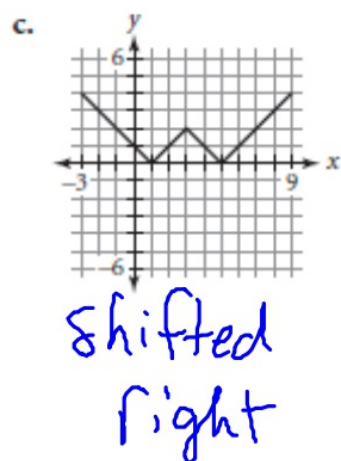
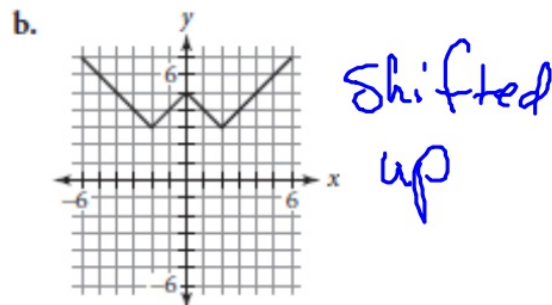
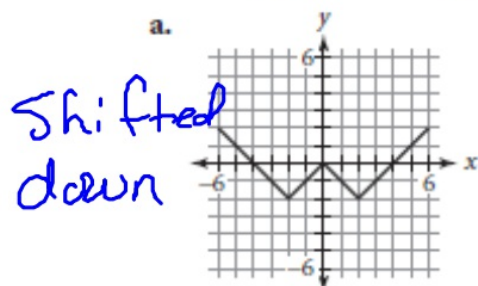
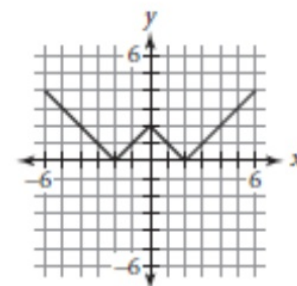


IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

## 3.11 Comparing and Analyzing Function Types

11/14/14

The graph of  $y = f(x)$  is shown at right. Write an equation for each related graph showing how the function has been translated.



Vocabulary 3.11.1 p. 19

Practice 3.11.2

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

### 3.11 Comparing and Analyzing Function Types

11/17/14

**Use properties of exponents and logarithms to solve logarithmic equations.**

$$\frac{\log a}{\log b} = \log_b a$$

Determine whether each equation is true or false.

T a.  $\log 45 = \log 5 + \log 9$

T c.  $\log_5 9 - \log_5 2 = \log_5 4.5$

F e.  $\log 12 - \log 4 = \log 8$   
 $\log 3$

F b.  $\log 8 = \frac{\log 32}{\log 4} = \log_4 32$

F d.  $\log 32 = \frac{1}{5} \log 2 = 5 \log 2$

f.  $\log \frac{1}{5} = \frac{1}{\log 5}$   
 $= \log 1 - \log 5$

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



## 3.11 Comparing and Analyzing Function Types

11/17/14

**From  
Friday**

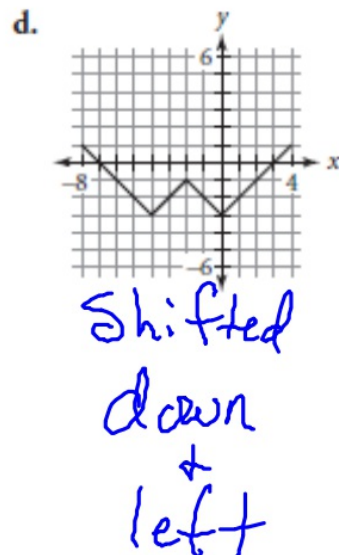
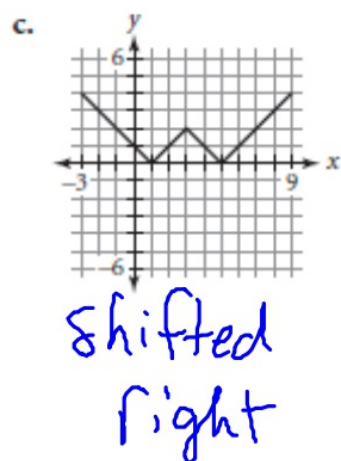
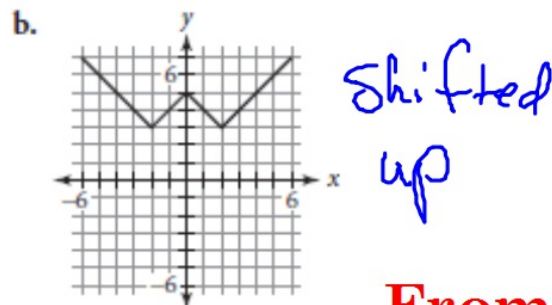
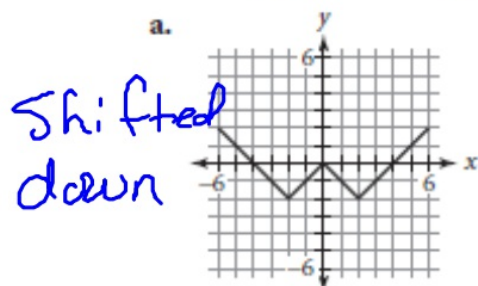
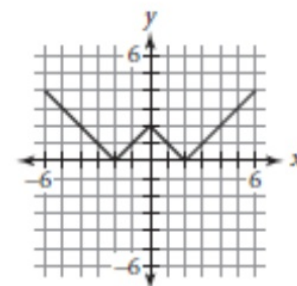
Transformation	Transforming the function	Transforming the table values	Transforming the graph
<b>Reflection over the line <math>y = x</math></b>	Switch the variables $x$ and $y$ in the equation.	Switch the $x$ - and $y$ -values in the table.	Switch the $x$ - and $y$ -values in the graph.
<b>Reflection over the <math>x</math>-axis</b>	Multiply the function by $-1$ : $-f(x)$ .	Multiply all $y$ -values in the table by $-1$ .	Flip the graph horizontally over the $x$ -axis.
<b>Reflection over the <math>y</math>-axis</b>	Substitute $-x$ for $x$ and simplify: $f(-x)$ .	Multiply all $x$ -values in the table by $-1$ .	Flip the graph vertically over the $y$ -axis.
<b><sup>1</sup>Horizontal shift, slide, or translation</b>	Substitute $x - k$ for $x$ and simplify: $f(x - k)$ .	Subtract a constant $k$ from the $x$ -values in the table.	Move the graph $k$ units horizontally.
<b><sup>2</sup>Vertical shift, slide, or translation</b>	Add a constant $k$ to the function itself: $f(x) + k$ .	Add a constant $k$ to the $y$ -values in the table.	Move the graph $k$ units vertically.
<b>Vertical stretch</b>	Multiply the function by a value greater than 1: $k \cdot f(x)$ .	Multiply all $y$ -values by a value greater than 1.	Stretch the graph along the $y$ -axis.
<b>Vertical compression</b>	Multiply the function by a value between 0 and 1: $k \cdot f(x)$ .	Multiply all $y$ -values by a value between 0 and 1.	Compress the graph along the $y$ -axis.

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

## 3.11 Comparing and Analyzing Function Types

11/14/14

The graph of  $y = f(x)$  is shown at right. Write an equation for each related graph showing how the function has been translated.



**From  
Friday**

Vocabulary 3.11.1 p. 19

Practice 3.11.2

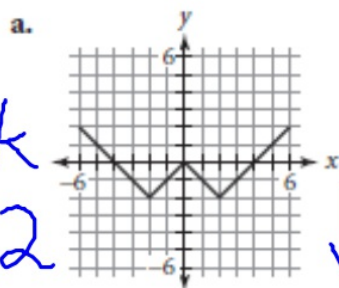
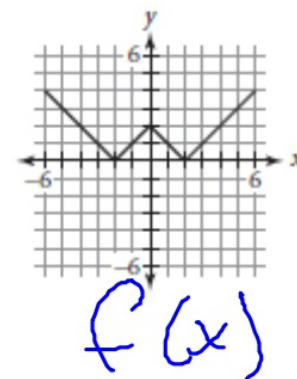
IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



### 3.11 Comparing and Analyzing Function Types

11/17/14

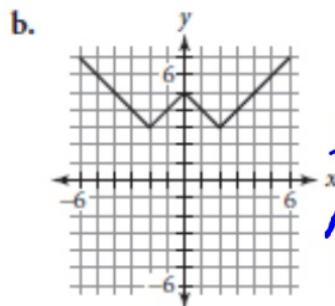
The graph of  $y = f(x)$  is shown at right. Write an equation for each related graph showing how the function has been translated.



$$f(x) - k$$

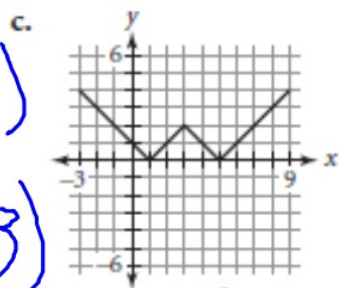
$$f(x) - 2$$

$\downarrow 2$



$$f(x) + 3$$

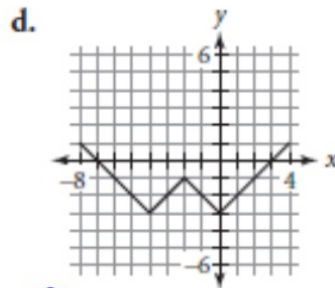
$\uparrow 3$



$$f(x - h)$$

$$f(x - 3)$$

$\xrightarrow{3}$



$$f(x + 2) - 3$$

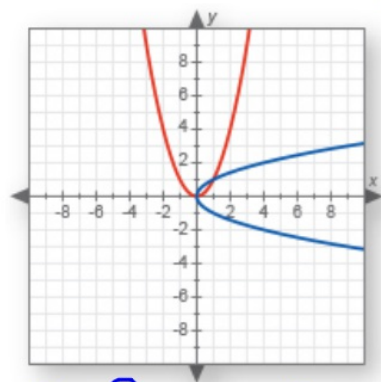
$\xleftarrow{2} \downarrow 3$

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

### 3.11 Comparing and Analyzing Function Types

11/17/14

Write the equation of the new function.

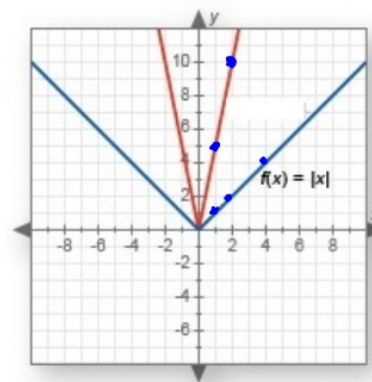


Reflection  
across  $y=x$

$$\underline{y = x^2}$$

$$3 \cdot 3 = 3^2$$

$$\underline{x = y^2}$$



Vertical stretch

$$y = |x|$$

$$y = 5|x|$$

$$y = |5x|$$

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

### 3.11 Comparing and Analyzing Function Types

11/17/14

What is the parent function of each and how has each function been transformed?

$$f(x) = 2 * 5^{(x+1)}$$

$$f(x) = 5^x$$

$$\leftarrow 1$$

Vertical stretch  
factor of 2

$$g(x) = - \left| \frac{x-3}{4} \right| + 1$$

$$g(x) = |x|$$

$$\uparrow 1 \quad \rightarrow 3$$

reflection across the  
x-axis

Vertical compression  
factor of  $\frac{1}{4}$

#### Practice 3.11.2

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

### 3.11 Comparing and Analyzing Function Types 11/18/14

**What qualifies as a function transformation?**

Reflection across  $y=x$ ,  $x$ -axis,  $y$ -axis

Shift - horizontally, vertically

Vertical stretch, compression

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



### 3.11 Comparing and Analyzing Function Types

11/18/14

Write the  $x$ - $y$  pairs and the function for each of the transformations.

$f(x)$	Shifted up 4	Shifted left 2	Flipped across the $y$ -axis																																								
<table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>2</td><td>5</td></tr> <tr><td>4</td><td>2</td></tr> <tr><td>7</td><td>8</td></tr> </tbody> </table>	$x$	$y$	0	1	2	5	4	2	7	8	<table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr><td>0</td><td>5</td></tr> <tr><td>2</td><td>9</td></tr> <tr><td>4</td><td>6</td></tr> <tr><td>7</td><td>12</td></tr> </tbody> </table>	$x$	$y$	0	5	2	9	4	6	7	12	<table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr><td>-2</td><td>1</td></tr> <tr><td>0</td><td>5</td></tr> <tr><td>2</td><td>2</td></tr> <tr><td>5</td><td>8</td></tr> </tbody> </table>	$x$	$y$	-2	1	0	5	2	2	5	8	<table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>-2</td><td>5</td></tr> <tr><td>-4</td><td>2</td></tr> <tr><td>-7</td><td>8</td></tr> </tbody> </table>	$x$	$y$	0	1	-2	5	-4	2	-7	8
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<table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr><td>-2</td><td>3</td></tr> <tr><td>0</td><td>-1</td></tr> <tr><td>2</td><td>2</td></tr> <tr><td>5</td><td>-4</td></tr> </tbody> </table>	$x$	$y$	-2	3	0	-1	2	2	5	-4	$-f(x + 2) + 4$																																
$x$	$y$																																										
-2	3																																										
0	-1																																										
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5	-4																																										

Reflected across the  $x$ -axis, shifted up 4 and left 2.

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



### 3.11 Comparing and Analyzing Function Types

11/18/14

**On your white board, with a partner apply the transformations to the given function.**  $f(x) = 3^x$

Transformations to apply:

- to the right 6  $f(x-6) = 3^{x-6}$
- up 4  $f(x)+4 = 3^x + 4$
- vertical stretch 3  $3f(x) = 3 \cdot 3^x$
- vertical compression 0.2  $0.2f(x) = 0.2 \cdot 3^x$
- reflected across the x-axis  $-f(x) = -(3^x)$
- reflected across the y-axis  $f(-x) = 3^{-x}$
- ALL of these together

$$-3 \cdot 0.2 f(-x-6) + 4 = -0.6 \cdot 3^{(-x-6)} + 4$$

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

### 3.11 Comparing and Analyzing Function Types

11/18/14

**On your white board, answer the following question.**

**If the logarithmic parent function,  $f(x) = \log_4(x)$  is vertically stretched by a factor of 8, reflected across the  $x$ -axis and then translated left 2 units, what is the resulting function?**

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

**Complete this problem from yesterday.**

**On your white board, answer the following question.**

**If the logarithmic parent function,  $f(x) = \log_4(x)$  is vertically stretched by a factor of 8, reflected across the  $x$ -axis and then translated left 2 units, what is the resulting function?**

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



### 3.11 Comparing and Analyzing Function Types

11/19/14

Here is how I worked it. Did I do it correctly?

Justify your answer. (Tell me how you know.)

On your white board, answer the following question.

If the logarithmic parent function,  $f(x) = \log_4(x)$  is vertically stretched by a factor of 8, reflected across the x-axis and then translated left 2 units, what is the resulting function?

$$8f(x) = 8\log_4(x) \quad -f(x) = -\log_4(x)$$

$$f(x-h) \quad f(x+2) = \log_4(x+2)$$

$$-f(8x-2) = -\log_4(8x-2)$$

$$-8f(x+2) = -8\log_4(x+2)$$

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.

## 3.11 Comparing and Analyzing Function Types

11/19/14

### Practice 3.11.2

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.



## 3.11 Comparing and Analyzing Function Types

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Practice 3.11.2

Apex quiz 3.11.3

IWBAT apply transformations to a variety of function families and identify and compare functions in differing representations.